

NGST SLEW ALGORITHM DESCRIPTION

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Introduction

In this memo, the slew trajectory algorithm implemented in the NGST integrated system simulation software program is described. This algorithm is used to generate the desired slew position, rate, and acceleration profiles for commanding a feedback/feedforward control system.

The algorithm objectives are to minimize the effect of the control torque on the flexible body motion and to minimize the maneuver time. With reaction wheels as control actuators, initial wheel speeds as well as individual wheel torque and momentum limits become additional constraints in the formulation. These constraints are necessary to ensure that unexpected saturation in reaction wheels can not occur during the slew, since such saturation can lead to control failure in tracking commanded motion, and can produce high frequency torque components capable of exciting structural modes.

There are two steps in the development of the algorithm. The first step involves the basic trajectory selection, which defines and parameterizes the functional form of the desired slew position, rate, and acceleration profiles. This step while using a similar approach to that described in [1], extends it by taking into account the reaction wheel limits in obtaining the desired motion profiles. The second step optimizes the slew command parameters, such as maximum slew acceleration, defined in the first step, for a given reaction wheel configuration, and torque and momentum limits, based on the dynamic interaction between the spacecraft and reaction wheel motion.

In [1], a near optimal maneuver trajectory algorithm is given for the case with no reaction wheel constraints. This algorithm specifies, in a parametric form, a smooth function that basically approximates the sign function of a bang-bang law, which is a well-known form for minimum-time controllers. The trajectory motion in this case is continuously maintained at maximum acceleration over a long portion of the time during the slew period. The duration of this acceleration time portion is selected in a trade-off between minimum maneuver time and the degree of smoothness in the trajectory.

In the presence of reaction wheel momentum and torque limits, arbitrarily long and continuously accelerating motions cannot be supported, implying that bang-bang controllers and the approach given in [1] are not applicable. With limits imposed on the reaction wheel momentum, the best that can be achieved in terms of minimizing maneuver time is to maintain spacecraft motion at its maximum rate allowable by these limits, over as long a portion of the slew time as possible.

Maximizing this coasting time period means minimizing the time it takes to reach the maximum rate at the start of the slew, and the time it takes to go back to zero rate at the end of the slew. This requires that the maximum allowable acceleration and de-acceleration, which are bounded by the reaction wheel torque limits and initial wheel momentum, be used in getting to and from the maximum rate.

Thus, the general acceleration profile appropriate for this case starts with an impulse whose amplitude is derived from reaction wheel torque limits, and whose direction is the direction of the desired motion. Following the impulse, the profile remains at zero over the entire slew duration, and ends with an impulse equal to the one applied at the start but in the opposite direction. The result in

the rate profile is simply a rectangular curve, whose amplitude is derived from the reaction wheel momentum limits.

In order to minimize the structural mode excitation, the selected acceleration profile is a smooth version of the impulsive profile described above, and is chosen so that it and its first derivative are continuous and given in low order polynomials. The basic polynomials used in [1] are employed to provide the smooth feature of the slew motion including those at the two end points.

Derivation

To achieve a smoothly varying torque, the desired acceleration profile generated by the slew trajectory algorithm is selected to have the following functional form:

$$\text{E.1} \quad \mathbf{a}(t) = \mathbf{a}_d \hat{e} \frac{d}{dt} f(t, \Delta t_a, \Delta t_c)$$

Where \mathbf{a}_d is the maximum acceleration amplitude, \hat{e} is a constant vector specifying the slew rotation axis, and $f(t)$ is a positive scalar time-function representing the desired spacecraft rate, parameterized by $(\Delta t_a, \Delta t_c)$, and whose first order derivative is given as:

$$\text{E.2} \quad \frac{d}{dt} f(t, \Delta t_a, \Delta t_c) = \begin{cases} \left(\frac{t-t_0}{\Delta t_a} \right)^2 \left(3 - 2 \left(\frac{t-t_0}{\Delta t_a} \right) \right) & t < t_1 = t_0 + \Delta t_a \\ \left(\frac{t_2-t}{\Delta t_a} \right)^2 \left(3 - 2 \left(\frac{t_2-t}{\Delta t_a} \right) \right) & t < t_2 = t_1 + \Delta t_a \\ 0 & t < t_3 = t_2 + \Delta t_c \\ \left(\frac{t-t_3}{\Delta t_a} \right)^2 \left(3 - 2 \left(\frac{t-t_3}{\Delta t_a} \right) \right) & t < t_4 = t_3 + \Delta t_a \\ \left(\frac{t_5-t}{\Delta t_a} \right)^2 \left(3 - 2 \left(\frac{t_5-t}{\Delta t_a} \right) \right) & t < t_5 = t_4 + \Delta t_a \\ 0 & t \geq t_5 \end{cases}$$

It follows that the desired rate profile has the following form:

$$\text{E.3} \quad \mathbf{w}(t) = \mathbf{w}_d \hat{e} f(t, \Delta t_a, \Delta t_c)$$

The profile of the slew angle \mathbf{Dq} can be directly obtained by integrating E.3:

$$\text{E.4} \quad \mathbf{Dq}(T) = \mathbf{w}_d \hat{e} \int_0^T f(t', \Delta t_a, \Delta t_c) dt'$$

Figure.1 shows the slew angle, rate, and acceleration profiles in the normalized form, with acceleration and rate magnitudes of 1, and the time periods, $\mathbf{Dt}_a=1$ and $\mathbf{Dt}_c=2$.

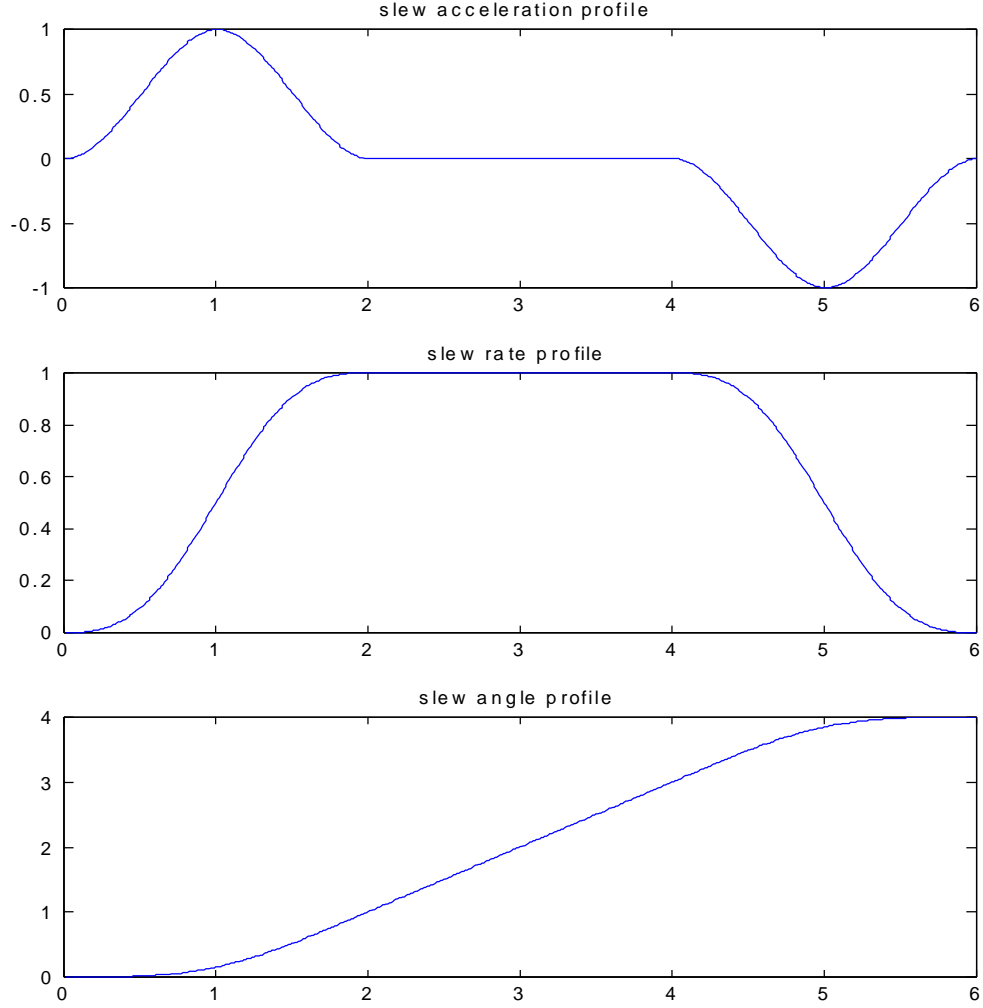


Figure. 1

With function $f(t)$ given above, it can be shown that in terms of the maximum acceleration magnitude \mathbf{a}_d , the maximum rate magnitude \mathbf{w}_d , and the slew angle \mathbf{Dq} , the time periods \mathbf{Dt}_a and \mathbf{Dt}_c can be expressed as:

$$\Delta t_a = \frac{\mathbf{w}_d}{\mathbf{a}_d}$$

E.5

$$\Delta t_c = \frac{\Delta \mathbf{q}}{\mathbf{w}_d} - 2 \frac{\mathbf{w}_d}{\mathbf{a}_d}$$

and the total slew period is:

E.6

$$\mathbf{DT}_s = \mathbf{DT}_c + 4 \mathbf{DT}_a$$

By imposing that a dead band zone in the acceleration profile, i.e. a zone where the time-derivative of $f(t)$ is zero, be maintained in all slew commands, even if it is just a point between transition from positive to negative acceleration, the maximum rate magnitude \mathbf{w}_d must also satisfy the following condition:

E.7

$$\mathbf{w}_d \leq \sqrt{\frac{\mathbf{a}_d \mathbf{Dq}}{2}}$$

which can be given in terms of \mathbf{DT}_a by using the first relation in E.5 for \mathbf{a}_d , i.e.:

$$2 \mathbf{DT}_a \leq \frac{\mathbf{Dq}}{\mathbf{w}_d}$$

and the second relation in E.5 leads to $\mathbf{DT}_c \approx \mathbf{0}$, which implies the presence of the dead band zone in the acceleration profile.

For later reference in the derivation of the maximum acceleration magnitude, given here are the relations between \mathbf{a}_d , \mathbf{w}_d , \mathbf{DT}_a , and the slew angle $\mathbf{Dq}(\mathbf{DT}_a)$ at time \mathbf{DT}_a after the start of the slew, when the desired acceleration profile reaches its positive maximum amplitude \mathbf{a}_d , and the desired rate profile reaches half of its maximum value \mathbf{w}_d :

$$\mathbf{a}_d = \frac{3}{20} \frac{\mathbf{w}_d^2}{\Delta \mathbf{q}(\Delta t_a)}$$

E.8

$$\Delta t_a = \frac{20}{3} \frac{\Delta \mathbf{q}(\Delta t_a)}{\mathbf{w}_d}$$

Derivation of the maximum rate magnitude

The basic equation describing the system momentum of the spacecraft and reaction wheels in the inertial frame is:

$$\text{E.9} \quad H_0 = R^T(t) \left(A_w^b h_w(t) + I_s \mathbf{w}(t) \right)$$

Where h_w is the wheel momentum vector in wheel frame, and A_w^b represents the constant transformation from the wheel frame to the spacecraft frame. In a four-wheel configuration, h_w is a 4-component vector, and A_w^b is a 3×4 matrix whose columns are reaction wheel rotational axes given in the spacecraft frame and can be obtained directly from the reaction wheel configuration. The left inverse of this matrix, denoted by \hat{A}_b^w , is a 4×3 matrix and typically given to minimize the wheel speed magnitude. H_0 and I_s are, respectively, the initial system momentum in the inertial frame, and the system inertia matrix in the spacecraft frame. H_0 is assumed here to be a constant vector, i.e. there is no external torque, and consists only of wheel momentum. $R(t)$ is the direction cosine matrix representing the transformation from the inertial frame to the spacecraft frame corresponding to the desired rate profile $\mathbf{w}(t)$.

Using the minimum mean squared solution, \hat{A}_b^w , for the inverse of A_w^b , the wheel momentum vector of minimum magnitude given in the reaction wheel frame, as a function of the initial system momentum and the desired motion profile can be written as:

$$\text{E.10} \quad h_w = \hat{A}_b^w \left(R(t) H_0 - I_s \mathbf{w}(t) \right)$$

For a redundant wheel configuration, where there are more than 3 wheels in operation for attitude control, if the null vector in the wheel frame is non-zero initially, then it must be added to E.10 and must be carried in all the subsequent steps in computing the maximum rate magnitude.

With E.3, E.10 can be expressed in terms of slew parameters as:

$$\text{E.11} \quad h_w = \hat{A}_b^w \left(R(t) H_0 - I_s \hat{\mathbf{e}}_d f(t) \right)$$

Let $\pm h_{\text{lim}}$ and $\pm \mathbf{t}_{\text{lim}}$, $h_{\text{lim}} > 0$ and $\mathbf{t}_{\text{lim}} > 0$, denote the momentum limits and torque limits, respectively, of each reaction wheel. These limits correspond to the four corners of a rectangular torque-speed curve. This is a simplified version of a more typical torque-speed curve, which has regions of decreasing torque as the wheel nears saturation speeds. Although it is a straightforward extension, only the rectangular torque-speed curve is considered here.

The momentum limits when applied to a reaction wheel leads to the following inequality:

$$\text{E.12} \quad -h_{\text{lim}} \leq h_{wi} = \left(\hat{A}_b^w \right)_i \left(R(t) H_0 - I_s \hat{\mathbf{e}}_d f(t) \right) \leq h_{\text{lim}}$$

Where the index i indicates the i^{th} wheel, and $(\hat{A}_b^w)_i$ denotes the i^{th} row of the body-to-wheel transformation matrix. E.12 leads to the following upper and lower bounds of the spacecraft rate component along the i^{th} wheel axis, due to the momentum limits of the i^{th} wheel:

$$\text{E.13} \quad -h_{\text{lim}} + \left(\hat{A}_b^w\right)_i R(t)H_0 \leq \left(\hat{A}_b^w\right)_i I_s \hat{e} \mathbf{w}_d f(t) \leq h_{\text{lim}} + \left(\hat{A}_b^w\right)_i R(t)H_0$$

Global bounds over the slew period can be obtained by taking the appropriate maximum and minimum values of the time-dependent terms over the period, i.e.

$$\text{E.14} \quad -h_{\text{lim}} + \text{Max}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w\right)_i R(t)H_0 \right\} \leq \left(\hat{A}_b^w\right)_i I_s \hat{e} \mathbf{w}_d f(t) \leq h_{\text{lim}} + \text{Min}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w\right)_i R(t)H_0 \right\}$$

The expression inside the minimum and maximum operators over the slew time period represents the component of the system momentum that the i^{th} wheel must be able to carry during the slew. As the spacecraft rotates, there are for each wheel two attitudes at which the system momentum on the wheel are maximum and minimum, which occur when the spacecraft frame is closest and farthest from the wheel's rotational axis. In addition the initial and final slew attitudes may represent extreme points. For the four wheel case, there are 10 of these rotation angles, and the exact solutions of these angles can be obtained since $R(\sphericalangle)$ can be expressed in terms of position without being explicitly dependent on time.

The computation of these attitudes are simplified by considering a coordinate frame, referred to as the slew frame in which the z-axis coincides with the slew axis, \hat{e} , and the x axis is defined such that the initial system momentum H_0 is in the x-z plane. In this frame, the H_0 describes a cone about the z axis as the slew progresses, and for a given wheel rotational axis vector, the minimum and maximum functions of E.14 can be computed straightforwardly. They can be shown to correspond to having H_0 at either the end points or lying in the plane spanned by the z-axis and the i^{th} wheel's rotational axis.

Since $f(t)$ is a positive function with the two end points at zero value, the upper and lower bounds on the body rate given in E.14 must have one bound positive and the other negative. Thus, the maximum rate amplitude based on the i^{th} wheel's momentum limit, denoted by \mathbf{w}_{di} , is simply the larger of the two bounds (i.e. the positive bound), i.e.:

$$\text{E.15} \quad \mathbf{w}_{di} = \text{Max} \left\{ \frac{-h_{\text{lim}} + \text{Max}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w\right)_i R(t)H_0 \right\}}{\left(\hat{A}_b^w\right)_i I_s \hat{e}}, \frac{h_{\text{lim}} + \text{Min}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w\right)_i R(t)H_0 \right\}}{\left(\hat{A}_b^w\right)_i I_s \hat{e}} \right\}$$

Also, the condition for a valid command slew is that the bounds defined in E.14 must have opposite signs. This assures that the component of the system momentum on a reaction wheel cannot exceed h_{lim} during the slew.

The sign of the denominator in E.15 plays a very important role in determining which of the two bounds is the maximum rate amplitude, as it indicates the direction in which that wheel must be rotated to achieve the desired spacecraft motion in the absence of the initial momentum. In general, it has the same sign as that of the wheel momentum limit that will be reached during the slew. In the case where this quantity is identically zero for the i^{th} wheel, there is no contribution from this reaction wheel to the slew and thus, it can be removed from the computation of the slew maximum rate and acceleration magnitudes.

The maximum rate amplitude \mathbf{w}_d that satisfies the momentum limits of all the reaction wheels in the system, is the smallest of all the \mathbf{w}_{di} :

$$\text{E.16} \quad \mathbf{w}_d = \text{Min}_i \{ \mathbf{w}_{di} \}$$

In the special case, where the initial system momentum H_0 is along the slew axis \hat{e} , E.13 for all the reaction wheels, simply reduce to:

$$-h_{\text{lim}} + \left(\hat{A}_b^w \right)_i H_0 \leq \left(\hat{A}_b^w \right)_i I_s \hat{e} \mathbf{w}_d f(t) \leq h_{\text{lim}} + \left(\hat{A}_b^w \right)_i H_0$$

which has constant bounds so that the maximum rate magnitude is easily obtained in this case.

Derivation of the maximum acceleration magnitude

The wheel torque required to carrying out the desired motion is obtained by taking the derivative of E.10, which gives:

$$\text{E.17} \quad \frac{d}{dt} h_w = \hat{A}_b^w \left([\mathbf{w}(t)] R(t) H_0 - I_s \frac{d}{dt} \mathbf{w}(t) \right)$$

Where the square brackets denote the matrix corresponding to the cross product operator, i.e. $[a]b = a \times b$.

With E.1 and E.3, E.17 can be rewritten in terms of slew parameters as:

$$\text{E.18} \quad \frac{d}{dt} h_w = \mathbf{w}_d \hat{A}_b^w [\hat{e}] R(t) H_0 f(t) - \mathbf{a}_d \hat{A}_b^w I_s \hat{e} \frac{d}{dt} f(t)$$

Using the same convention as in the above section in expressing components related to a reaction wheel, the following inequality is obtained when applying the torque limits to the i^{th} reaction wheel:

$$\text{E.19} \quad -\mathbf{t}_{\text{lim}} \leq \frac{d}{dt} h_{wi} = \mathbf{w}_d \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 f(t) - \mathbf{a}_{di} \left(\hat{A}_b^w \right)_i I_s \hat{e} \frac{d}{dt} f(t) \leq \mathbf{t}_{\text{lim}}$$

The index i appears in \mathbf{a}_{di} to indicate that the derived parameter in this case is the maximum allowable acceleration due to the i^{th} wheel. Rearranging E.19 leads to the following upper and lower bounds on the component of the spacecraft acceleration along the i^{th} wheel axis, due to the torque limits on that wheel:

$$\text{E.20} - \mathbf{t}_{lim} + \mathbf{w}_d \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 f(t) \leq \mathbf{a}_{di} \left(\hat{A}_b^w \right)_i I_s \hat{e} \frac{d}{dt} f(t) \leq \mathbf{t}_{lim} + \mathbf{w}_d \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 f(t)$$

At this point, an approach similar to what was used previously to derive the maximum rate magnitude, can be used to obtain an legitimate maximum value for the acceleration profile. This involves obtaining the global bounds for the time-dependent term in E.20, which is the gyroscopic effect term, over the slew period, and taking the minimum of the norm of the two bounds as the maximum allowable acceleration by the i^{th} wheel. The desired acceleration magnitude is simply the minimum of these maximum allowable acceleration values over all wheels, i.e.

$$\mathbf{a}_{di} = \text{Min} \left\{ \left| \frac{-\mathbf{t}_{lim} + \mathbf{w}_d \text{Max}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 \right\}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right|, \left| \frac{\mathbf{t}_{lim} + \mathbf{w}_d \text{Min}_{t \in \Delta T_s} \left\{ \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 \right\}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right| \right\}$$

$$\text{and } \mathbf{a}_d = \text{Min}_i \{ \mathbf{a}_{di} \}.$$

This is an admissible solution because as the acceleration profile, which is a function of the derivative of function $f(t)$ given in E.2, changes sign from positive to negative in the course of the slew, the norm of the smaller bound will ensure that the acceleration profile will not exceed torque limit of the i^{th} wheel. By taking the minimum over these smaller bounds of all the reaction wheels, the acceleration profile will be within all the wheel torque limits. Although it produces a legitimate solution for the maximum acceleration magnitude, and is actually quite easy to implement, this approach in general results in sub-optimum acceleration profile.

An approach, which yields an optimum solution for the maximum acceleration amplitude, derives the maximum acceleration values for the positive and negative regions of the acceleration profile separately, and selects the minimum of the two as the solution. This approach turns out to be more complicated to implement since it requires an iterative method for finding roots of an equation involving transcendental functions.

The inequality given by E.20 when applied to the positive region of the acceleration profile reduces to:

$$\text{E.21} \quad 0 \leq \mathbf{a}_{di}(+) \frac{d}{dt} f(t) \leq \left| \frac{\mathbf{t}_{lim}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right| + \frac{\mathbf{w}_d \left(\hat{A}_b^w \right)_i [\hat{e}] R(t) H_0 f(t)}{\left(\hat{A}_b^w \right)_i I_s \hat{e}}$$

By evaluating the above inequality at time $\mathbf{D}t_a$ the following inequality is obtained:

$$\text{E.22} \quad \mathbf{a}_{di}(+) \leq \left| \frac{\mathbf{t}_{lim}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right| + \frac{\frac{1}{2} \mathbf{w}_d \left(\hat{A}_b^w \right)_i [\hat{e}] R(\mathbf{D}t_a) H_0}{\left(\hat{A}_b^w \right)_i I_s \hat{e}}$$

This inequality contains two unknown parameters, $\mathbf{a}_{di}(+)$ and $\mathbf{D}t_a$, which, when obtained by solving the equation part of E.22, are the optimum allowable acceleration and the shortest time to reach the maximum rate magnitude, satisfying the system torque constraints and relations in E.5 for the i^{th} wheel in the positive acceleration region. Although it is possible for the equation in E.22 to have more than one set of solutions, the set which is the desired solution has the largest $\mathbf{a}_{di}(+)$ and smallest $\mathbf{D}t_a$.

Using E.8, the equation part of E.22 can be rewritten in terms of just one variable, the slew angle $\Delta\theta_i$ evaluated at $\mathbf{D}t_a$, for the i^{th} wheel:

$$\text{E.23} \quad \frac{\frac{3}{20} \mathbf{w}_d^2}{\Delta \mathbf{q}_i(\Delta t_a)} = \left| \frac{\mathbf{t}_{\text{lim}}}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}} \right| + \frac{\frac{1}{2} \mathbf{w}_d \left(\hat{\mathbf{A}}_b^w\right)_i [\hat{\mathbf{e}}] R(\Delta \mathbf{q}_i(\Delta t_a)) H_0}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}}$$

By using the slew coordinate frame defined in the previous section, E.23 can be simplified to the following form:

$$\text{E.24} \quad \frac{A}{\Delta \mathbf{q}_a} = B + C \sin(\Delta \mathbf{q}_a + \mathbf{f})$$

Where $\Delta\theta_a \equiv \Delta\theta_i(\Delta t_a)$ is the slew angle at which the acceleration profile attains its positive maximum magnitude \mathbf{a}_d and the rate profile attains half of its maximum magnitude \mathbf{w}_d . A , B , C , and \mathbf{f} are constants corresponding to terms in E.23, i.e.

$$A = \frac{3}{20} \mathbf{w}_d^2, \quad B = \left| \frac{\mathbf{t}_{\text{lim}}}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}} \right|, \quad C = \frac{\frac{1}{2} \mathbf{w}_d \|H_0\| S_1\left(\left(\hat{\mathbf{A}}_b^w\right)_i, H_0, \hat{\mathbf{e}}\right)}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}}, \quad \mathbf{f} = \text{atan}\left(S_2\left(\left(\hat{\mathbf{A}}_b^w\right)_i\right)\right)$$

With $S_1(\cdot)$ and $S_2(\cdot)$ are scalar functions given in terms of specified parameters, with S_1 assumed values between (-1,1), which limits the possible size of C . The terms of E.24 can be visualized as follows: B is the maximum possible acceleration that the i^{th} wheel can provide along the slew rotation axis $\hat{\mathbf{e}}$. The term $C \sin(\Delta \mathbf{q}_a + \mathbf{f})$ is the acceleration required to commute the system momentum through the wheels as the slew progresses. The left-hand side of E.24 is the remaining acceleration available to slew the body.

Although E.24 can not be solved analytically for $\Delta\theta_a$, some simple iterative search methods can be devised for finding the smallest root, which corresponds to the desired (minimum slew time) solution.

One adequate search technique to find the first root is to start at the top left of the region, which corresponds to the zero angle and acceleration ($B + |C|$). The angle corresponding to this point using

the left-hand side of E.24, i.e. $\mathbf{q}_0 = \frac{A}{B + |C|}$, should be within the bounded region and on the left of

the first root. Starting with this angle, by stepping in an adequately small increment along the curve of the left-hand side of E.24, the interval, which contains the solution, can be determined. The following iteration method can be shown to yield the desired solution within this interval:

$$\text{E.26} \quad \left\{ \mathbf{q}_k = \frac{A}{\hat{\mathbf{a}}_{k-1}} < \mathbf{q}_{\max} \Rightarrow \hat{\mathbf{a}}_k = B + C \sin(\mathbf{q}_k + \mathbf{f}) \right\} \Rightarrow \hat{\mathbf{a}}_i \rightarrow \mathbf{a}_{di}(+)$$

However, at any point in the above process, if an angle $\mathbf{q}_i > \mathbf{q}_{\max}$, where \mathbf{q}_{\max} is the angle defines the boundary of the first quadrant of the total slew angle, the maximum rate magnitude must be decreased, according to E.7, to give a smaller A that will permit the above process to continue. This rate reduction is simply to adjust the desired motion profile so that available reaction wheel torque in the system can allow the motion to accelerate to the maximum rate within the time constraint to have a dead band in the acceleration profile.

The same method described above can be applied to the negative region of the acceleration profile to obtain the desired solution for the negative region. The equivalent equation to E.23 for the negative region of the acceleration profile is the following:

$$\text{E.27} \quad \frac{\frac{3}{20} \mathbf{w}_d^2}{\mathbf{Dq}(\mathbf{Dt}_s) - \mathbf{Dq}(\mathbf{Dt}_s - \mathbf{Dt}_a)} = \left| \frac{\mathbf{t}_{\lim}}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}} \right| - \frac{\frac{1}{2} \mathbf{w}_d \left(\hat{\mathbf{A}}_b^w\right)_i [\hat{\mathbf{e}}] R(\mathbf{Dq}(\mathbf{Dt}_s - \mathbf{Dt}_a)) H_0}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}}$$

Having obtained the maximum allowable acceleration values for each of the reaction wheels in the positive, $\mathbf{a}_{di}(+)$, and negative, $\mathbf{a}_{di}(-)$, regions, the maximum acceleration magnitude is the minimum value taken over all these values, i.e.

$$\mathbf{a}_d = \text{Min}_i \{ \mathbf{a}_{di}(-), \mathbf{a}_{di}(+) \}$$

In the special case, where the initial system momentum H_0 is along the slew axis $\hat{\mathbf{e}}$, and for wheel i^{th} which has term $\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}$ non-zero, the properly modified version of E.23 and E.27 leads to the following solution:

$$\mathbf{a}_{di} = \frac{3}{20} \min \left\{ \frac{\mathbf{w}_d^2}{\left| \frac{\mathbf{t}_{\lim}}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}} \right| + \frac{\frac{1}{2} \mathbf{w}_d \left(\hat{\mathbf{A}}_b^w\right)_i [\hat{\mathbf{e}}] H_0}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}}}, \frac{\mathbf{w}_d^2}{\left| \frac{\mathbf{t}_{\lim}}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}} \right| - \frac{\frac{1}{2} \mathbf{w}_d \left(\hat{\mathbf{A}}_b^w\right)_i [\hat{\mathbf{e}}] H_0}{\left(\hat{\mathbf{A}}_b^w\right)_i I_s \hat{\mathbf{e}}}} \right\}$$

\mathbf{w}_d and \mathbf{a}_d computed from the above formulation based on reaction wheel configuration, momentum and torque limits, and slew command, completely define the desired slew trajectory.

Reference:

[1]- J.L.Junkins, et. al. "Near-Minimum-Time Control of Distributed Parameter Systems: Analytical and Experimental Results", Journal Guidance & Control Vol. 14 No.2 ,1990.